THE BEHAVIOR OF VELOCITY ENHANCEMENT IN MICROCAPILLARY FLOWS OF FLEXIBLE WATER-SOLUBLE POLYMERS

YOUNG HYO SEO, O OK PARK, AND MYUNG-SUK CHUN

Department of Chemical Engineering, Korea Advanced Institute of Science and Technology, 373-1, Kusung-dong, Yuseong-gu, Taejon 305-701, South Korea

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A concentration depletion is developed nearby a capillary wall from the result of flow-induced radial migration to the axis region of capillary incorporating with the chain extension. The velocity enhancement factor can be defined quantifying the migration of polymers. For the experimental observation of the radial migration, the capillary hydrodynamic fractionation (CHDF) system has been adopted as a probe on the micropipet flow of water-soluble terrylene polymers. As a practical implication, the migration phenomena may lead to apply this system due to the possible alteration of the elution characteristics. In order to verify the experimental results more accurately, a model has been provided based on the diffusion equation and the conformation-dependent friction involving a radial dependence of hydrodynamic force on the dumbbell extension. For the nonionic polycarboxylic acid, the experimental behavior of velocity enhancement was shown to be in accordance with the present model within the somewhat increased We. In case of the anionic polyelectrolyte with 30% hydrolyzed polycarboxylic acid, the increasing slopes of velocity enhancement are different with the eluent ionic strength. Note that the experimental data allow us to consider an enhanced migration due to the long-ranged repulsive force between capillary wall and polymer.

Introduction

Recently, Chun et al. (1993) have presented the analysis on the development of concentration-depleted layer and the corresponding experimental results for semiflexible polysaccharide using capillary hydrodynamic fractionation (CHDF) system. Their results showed that, when the eluant velocity is low enough, the velocity enhancement factor of dilute xanthan polyelectrolyte is decreased as the eluant ionic strength increases. For a higher eluant velocity, however, the prediction loses its accuracy due to the hydrodynamic force effect on xanthan motion. They suggested the further work regarding the hydrodynamic effect on a concentration profile and a velocity enhancement. In the concentration profile, in origin, a flow-induced inhomogeneous distribution generates hydrodynamic thickening of the depleted layer in the vicinity of capillary wall. Apart from its substantial academic interest, implications of the concentration depletion principle in size and surface exclusion chromatographies, flows in porous membranes, and related large process in enhanced oil recovery can be significant.

* Received on December 11, 1995. Correspondence concerning this article should be addressed O. Ok Park.
** Present Address: Dept. of Chem. Eng. and Materials Sci. University of California, Davis, CA 95616, USA

The presence of a stress field perturbs the average chain dimension, elongating and orienting the polymer chain in the direction of flow field. The behavior of flow-induced radial migration in the confined space of narrow channel incorporated with the concentration depletion, has been studied by many scientists long times ago. Although it has been identified to be erroneous later, Terrell and Malone (1977) modeled initially a polymer molecule as a simple Rouse bead and spring collection. There is a well-known thermodynamic driving force tending to restore the unperturbed dimension of molecules. They get the chemical potential as a function of the shear rate and the molecular relaxation time of the flexible polymer with a constant friction. Reasonable speculation within about the last decade has been proposed that this driving force will cause polymer molecules to diffuse from region of high stress to region of low stress. With hydrodynamic approach, Sekhon et al. (1982) and Brunn (1983) have examined the migration of Rouse bead-spring molecules. It was shown conclusively that by accounting for the finite distance between beads and non-zero bead volumes, a hydrodynamic force leads to the migration across streamlines in Poiseuille flow.

Regarding another complicated behavior of concentration, it has been claimed that the thickness of the concentration-depleted layer increases at sufficiently high shear rates, although that is not easy...
to confirm experimentally. If we can employ very narrow capillary, however, this complicated feature can be possible to observe. Both the slip and concentration profiles have been demonstrated by Cehcn and Metzner (1982, 1988), who studied rigorously the effect on the radial migration of flexible polymer. Dutta et al. (1987) have performed the numerical work concerned with the polymer migration in a developing flow, continued from their previous analysis on the fully developed case for capillary flows. As a relevant experimental study, Park et al. (1992) have reported the analysis on the flows of partially hydrolyzed polyacrylamide (HPAM) and xanthan solutions through narrow capillaries with range of 100 to 250 μm. The depleted layer thickness of HPAM, which was much larger than the polymeric length scale, was shown to increase with increasing wall shear stress. In recent, the current status on the development of the origin of polymer migration phenomena has been fairly reviewed by Agarwal et al. (1994). Further, they discussed the practical implications, including various experimental observations. In order to quantify the concentration depletion from the polymer migration, the velocity enhancement factor in capillary flows can be observed. This factor has been early provided by Lecourier and Chauvetot (1984) and Hoagland and Prudhomme (1988, 1989), in which they experimentally observed the size fractionation of polymers with packed column by defining a retention factor. Their studies were stimulated on the role of surface exclusion and the flow strength effect. Note that Sorbie (1990) extended the two-fluid approach by considering the effects of the concentration-depleted layer on rheological and transport properties, although he treated the rigid rod-like polymer.

The purpose of the present study is to examine the velocity enhancement and associated flow-induced migration of flexible polymers in capillary flows. The CHDF system is applied here as a probe on the modification of concentration-depleted profiles through the detection of velocity enhancement factor in microcapillary columns. In this context, our experiments could offer another feasible technique since the other methods, such as evanescent wave-induced fluorescence (EWIF), or X-ray reflectancence, have been developed to monitor the change of the depleted layers alone. Both the polycrylamide (PAAm) and the HPAM are selected as flexible water-soluble polymers. In case of the anionic HPAM polyelectrolyte, the ion-exclusion effect due to a repulsive force can be encountered in the flow behavior, where the velocity enhancement is clearly sensitive to the eluant ionic strength. In Discussion, we take into account the conformation-dependent friction model originally proposed by Fuller and Leal (1981), so as to verify our experimental results of the flexible chain behavior. Our model prediction is compared with the experimental measurements based on which we shall point out subsequently the distinct conclusions.

1. Capillary Flow Experiments

1.1 CHDF system

The CHDF apparatus has been designed as shown in Fig. 1. Originally, the CHDF technique has been developed as an improved size analysis for spherical particles (Doe-Ramos and Silebi, 1989, 1990). In addition to shorter analysis time the microcapillary column offers the advantage of a lower dispersion. The fused silica capillaries used were commercially available from Polymicro Technologies Inc. (Phoenix, AZ), which have inner diameters of 11, 25, and 75 μm with 375 μm in outer diameter. We considered that injected sample polymers should have enough residence time in the capillary to obtain a fully developed concentration profile, for which the required capillary length can be estimated. Sufficient negative charges are developed on the capillary wall due to the ionization of silanol groups in an aqueous solution. The capillary is connected in the splitting tee using a graphite ferrule which crushed onto the capillary. As an eluant, the aqueous solvent (HPLC grade) contains the bactericide of 5 mM formaldehyde. NaCl of 1-1 type electrolyte was chosen as an...
added ion source to the eluant to examine the ionic strength effect on the velocity enhancement, and total ionic strength of eluant was determined by the NaCl concentration. The eluant is delivered by ISCO microprocessor-controlled HPLC pump (Model 2350) equipped with an in-line pulse damper through a 10 μm inlet-filter from a reservoir. The polymer sample is loaded and injected into the microcapillary column by Valco injection valve having the sample loop of 10 μl. The splitting ratio is controlled by miniature metering valve, and the average eluant velocity is measured with the tracer of a molecular size solute, sodium dichromate.

To circumvent the problem of excessive peak broadening in the detector cell and in the capillary/detector connection, an on-column detection method was employed (Sugarman and Prud’homme, 1987), (Tijssen et al., 1986). By burning off a small portion of polyimide coating of capillary and subsequent cleaning the surface with acetone, this transparent section can be used as the detector cell for UV/VIS detection of eluting sample zone. Variable wavelength UV/VIS detector (CV′e absorbance detector, Varian, Cary3) fitted with a 194 nm wavelength was utilized to continuously monitor the turbidity. Maximum absorbance can be obtained in this wavelength. The output signal from the detector was monitored on a Hewlett-Packard integrator (Model HP394A) for data processing. The velocity enhancement factor χ can be experimentally detected as

\[ \chi = \frac{\text{eluant} \text{ tracer elution time}}{\text{polymer elution time}}. \]  (1)

Our experiments should be done in the dilute regime to eliminate a viscous effect on the fractogram shape or elution time. The dilute condition corresponds approximately to the below of critical overlap concentration C (=0.77/intrinsic viscosity) given by Grassley (1980). For very low concentration of over-limited by the detection limit, however, it is expected that the peak cannot be detected.

1.2 Flexible model polymers

The powdered samples of PAAm and partially HPAM were purchased from Aldrich Co. (MO, USA), and the supplier states that the HPAM has about 30% of the amide groups hydrolyzed to anionic carboxyl groups. We can also observe the carboxyl substitutions by examining the IR spectrum peaks. The properties of model polymers are presented in Table 1. The sample powders dried completely were dissolved by using a magnetic stirrer with a low speed for preventing possible mechanical degradation. Care was taken in sample preparation to remove any solid particles or microgels, for which all samples were filtered through a 3 μm cellulose ester membrane (Gelman Science). Measurements of an aqueous

<table>
<thead>
<tr>
<th>Polymer</th>
<th>(M_w \times 10^4)</th>
<th><a href="cm%C2%B2/g">η</a></th>
</tr>
</thead>
<tbody>
<tr>
<td>PAAm</td>
<td>5.5</td>
<td>1200⁹⁷</td>
</tr>
<tr>
<td>HPAM</td>
<td>0.2</td>
<td>389⁹⁷</td>
</tr>
</tbody>
</table>

a) Based on the Mark-Houwink equation with the condition of high ionic strength (0.1 M NaCl solution)
b) \([\eta]=4.9\times10^{-3}M_w^{0.6}\); Ref., Klein and Conrad (1980)
c) \([\eta]=3.1\times10^{-2}M_w^{0.7}\); We determined from the fittings based on the data of Tam and Tsu (1989) and Park et al. (1992)

![Fig. 2 Viscosity as a function of shear rate for nonionic PAAm and anionic HPAM (their molecular structures are also provided) at 25°C, each concentration being 1000 ppm](image)

PAAm solution were carried out within one week after the preparation owing to the time-dependence of solution properties. Sample solutions for the injection were prepared from the 1000 ppm stock solution by dilution.

Rheological properties were measured by a rotation viscometer (Physica-Rheolab MC20/UM/TEZ 180, Stuttgart, FRG) using a Z1 DIN cell system, which consists of bob and cup fixtures. Figure 2 exhibits the different behavior of ionic strength effect between nonionic PAAm and anionic HPAM samples. The HPAM in pure water shows shearthinning characteristics to some extent in high shear rate. There is a considerable decrease in the viscosity when 0.1 M aqueous NaCl solution is used as a solvent. For nonionic PAAm solutions, however, the addition of salt does not affect the shear viscosity. The presence of Na⁺ ion reduces the interaction among the negative charge presenting along the anionic polymer chain, and thus results in a reduction...
2. Experimental Results

Microcapillaries with inner diameters of 11 and 25 μm were employed for studying the effect of shear rate on the velocity enhancement factor of nonionic PAAm, in which the fully developed condition was satisfied. The PAAm concentration of 100 ppm experimented was dilute below the critical overlap concentration. The experimental result shown in Fig. 3 provides the increase of velocity enhancement factor in PAAm as the wall shear rate is increased, which is a direct experimental evidence of the flow-induced radial migration.

The velocity enhancement factor of 100 ppm HPAM with different eluent ionic strength shown in Fig. 4 was obtained with the capillary having inner diameter of 75 μm. As we verified previously with xanthan (Chun et al., 1993), the velocity enhancement factor of anionic HPAM increases as the ionic strength decreases. What we want to point out here is that the velocity enhancement factor increases more sharply as salt added. Figure 5 replotted the Fig. 4 illustrates the features of the ionic strength dependence of velocity enhancement. Note that this dependence arises, primarily, from the change in the long-ranged repulsive force between the capillary wall and the charged polymer. Although the velocity enhancement factor is plotted for narrow range of wall shear rate due to the limited data given in Fig. 4, it is clear that the velocity enhancement is independent of the shear rate for sufficient low shear rate. On the contrary, the vanishment of the velocity enhancement dependence on ionic strength can be expected ultimately for sufficient high shear rate. It is well known that the polyelectrolyte conformation like as partially HPAM is strongly dependent on the solvent ionic strength (Ait-Kadi et al., 1987), (Tam and Tiu, 1989, 1990). The intrinsic viscosity increases with decreasing ionic strength, indicating an extension of the polymer chain accompanied with an increase in the stiffness. The ionization of negative charge tends to retard the relaxation after polymer chains are extended, as a result, the observed effect should be an
increase of the relaxation time. However, the HPAM chain becomes nearly coil-like having flexible chain properties with an increase of ionic strength. This fact suggests that the velocity enhancement shown in Fig. 4 is closely related to the chain flexibility.

3. Discussion

3.1 Model description on the velocity enhancement

3.1.1 Potential with conformation-dependent friction
dumbbell

By considering the relevant rheological model a proper comparison with experimental data can be attempted. Since the shear rate in a fully developed Poiseuille flow is varied with the radial position, we suppose that the polymer chain along the streamlines at various radial positions will experience a non-uniform deformation resulting in a free energy gradient. A thermodynamic force tends to cause the diffusion of deformed polymers to the region of capillary axis. Incorporating the concentration gradients in flow fields, the entropic potential $f$ for the extension of Gaussian chain is given as (Flory, 1953)

$$f = kT \left[ \frac{a_i a_i - 3}{2} - \ln (a_i a_i a_i) \right]$$

(2)

Here, $a_i = \langle \chi_i^2 \rangle$, $i = 1, 2, 3$, $\langle \chi_i^2 \rangle$ is the mean square projection of segment position on $X_i$ axis, a subscript $i$ refers to the unperturbed state, and $kT$ is the Boltzmann temperature. The average property $\langle x_{xy} \rangle$ can be obtained by

$$\langle x_{xy} \rangle = \int x x_{xy} F(x, t) dx$$

(3)

Here, $F(x)$ is the configurational distribution function of the chain.

Flexible linear polymers can be described by an Hookean dumbbell model (Bird et al., 1987), in which two centers of hydrodynamic resistance are connected by a frictionless linear spring, and the origin of the spring force is the conformational entropy. A force balance on the separation vector $x_i$ which links the position vectors of the two beads with friction factor $\zeta$ is substituted into a conservation equation for the $x_i$. In general, the friction factor of highly extended chain differs from that of the random coil state with $\zeta_0$. Hinch (1977) suggested that the hydrodynamic friction force can be obviously explained by a dumbbell with conformation-dependent friction factor

$$\zeta = \zeta_0 Q(x)$$

(4)

where the ratio of friction factor $Q(x) = \sqrt{N} \langle x^2 \rangle^{1/2}$,
becomes extended considerably toward the wall far from the center region of capillary, and the friction factor exhibits a consequent increase.

3.1.2 Radial concentration profile and velocity enhancement factor

The radial flux $J_r$ due to the chemical potential gradient is balanced by the diffusion flux for a polymer concentration $C$ (Cohen and Metzner, 1986). From the analytical solution, significant concentration variations with the position are predicted at full development. A radial concentration distribution has the highest concentration at the capillary axis with the lowest shear rate, whereas the lowest concentration at the capillary wall. By introducing the constitutive diffusion theory conducted earlier (Cohen and Metzner, 1982), (Dutta et al., 1987), the diffusion equation in cylindrical coordinates can be obtained via the continuity equation as

$$\frac{\partial C}{\partial t} + \nabla \cdot CV = -\nabla \cdot J$$  \hspace{1cm} (6)

where $\mathbf{V}$ is the velocity vector. Assuming negligible both radial convection (i.e., $V_r \frac{\partial C}{\partial r} < V_z \frac{\partial C}{\partial z}$) and axial diffusion in steady-state (Cohen and Metzner, 1986), Eq. (6) reduces to

$$V_z \frac{\partial C}{\partial z} = -\frac{1}{r} \frac{\partial (r J_r)}{\partial r}$$  \hspace{1cm} (7)

Next, the radial flux can be given with the first order approximation for ideal polymer solution as follows

$$J_r = -D \left( \frac{\partial C}{\partial r} \right) - CE \left( \frac{\partial \beta}{\partial r} \right)$$  \hspace{1cm} (8)

where $D$ is the polymer mass diffusivity in solution, that is equivalent to the Fickian diffusion coefficient. The second term in right hand side of Eq. (8) is the potential due to the presence of an inhomogeneous stress field. The deformation and possibly orientation of polymer chain result in a decrease in the entropy, or equivalently to an increase in the free energy of the polymer.

The diffusion equation of Eq. (8) simplifies by requiring that at full development the net flux $J_r$ should be zero. We introduce the following dimensionless variables

$$C^* = \frac{C}{C_0}, \quad V_r^* = \frac{V_r}{\langle V_r \rangle}, \quad J_r^* = \frac{J_r}{f k T}$$  \hspace{1cm} (9)

where $C_0$ is the initial polymer concentration at capillary entry, $\langle V_r \rangle$ the average fluid velocity at the capillary axis, and $R$, the capillary inner radius. These are specified as the experimental parameters.

Although the viscosity of polymer solution is generally a function of both local shear rate and concentration, however, a Newtonian flow can be assumed reasonably by because the considered solution including the $C_1$ is so dilute. We then arrive at the following equation as a simple form

$$\frac{\partial C^*}{\partial r^*} + C^* \frac{\partial J_r^*}{\partial r^*} = 0$$  \hspace{1cm} (10)

Suppose we have $C_0$ and flow rate, and by mass balance, the mass flux of polymer across any plane through the capillary must be equal to the injected mass flux (Sorbie, 1990). That acts as a boundary condition expressed in integral type

$$\int_0^1 V_r^* C^* \, dr^* = \frac{1}{2}$$  \hspace{1cm} (11)

With the parabolic velocity profile, the solution of Eq. (10) is easily derived as

$$C^*(r^*) = \frac{\exp(-f^*)}{4\int_0^1 \exp(-f^*) \, r^*(1-r^2) \, dr^*}$$  \hspace{1cm} (12)

Figure 7(a) provides the We number dependency of the chemical potential, and then the radial concentration profile based on the bulk concentration for any We can be strictly determined from the computed $f^*$. Figure 7(b) demonstrates that as the $We$ increases, more and more of the polymers are forced to reside in the region near the capillary axis. In this, we can readily expect the concentration-depleted layer becomes larger with increasing of $We$.  

![Figure 7](image-url)

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Once the concentration profile is known, then it is straightforward to compute the velocity enhancement factor $\chi$. The polymers flowing with radial migration elute faster than the eluant fluid, due to the concentration depletion close to the capillary wall. As treated in the previous study (Chun et al., 1993), the $\chi$ is defined as follows

$$\chi = \frac{\langle V_r \rangle}{\langle V \rangle} = \frac{\int_0^2 \frac{C(r^*) V_r(r^*)}{r^*} r^* dr^*}{2\int_0^2 \frac{C(r^*)}{r^*} r^* dr^*}$$  \hspace{1cm} (13)

The flow-induced migration occurs when the polymer chain is subjected to the hydrodynamic forces greater than the forces of chain relaxation arising from Brownian motion. This can be explained by the fact that the $\chi$ is eventually a function of $We$ number. Also, it is important to note that the chain extension is apparently hindered by hydrodynamic friction forces with increasing the flow strength.

### 3.2 Comparison with the experimental results

The radial migration depends on both the shear rate and relaxation time of polymer, therefore, the relaxation time is required to provide the model prediction. The relaxation time is taken to be that of an elastic dumbbell, which is approximately equivalent to the sum of the relaxation times for each of the normal modes of Rouse-Zimm model

$$\lambda = \Omega \left[ \frac{c}{\eta_s M_s} \right] R_0 T$$  \hspace{1cm} (14)

in which $R_0$ is the gas constant, and $\eta_s$ and $[c]$ are the solvent viscosity and solution intrinsic viscosity respectively. The $\Omega$ is a constant of order 1, and it is given as 0.42 for random coil molecules in dilute solutions (Zimm, 1956). With the relaxation time estimated as 1.1 nsec, the behavior of velocity enhancement for FAAnM can be examined as shown in Fig. 8. The polymer velocity is equal to the fluid velocity in the low $We$ limit, however, the velocity enhancement factor begins to rise as the $We$ increases. It is clear that all the data obtained with both capillaries of different inner diameters fall on a single curve, showing an internal consistency, which confirms a shear rate is an important parameter governing the extent of velocity enhancement for given polymer.

Note that in a weak flow condition (generally, $We<1$) only a surface-exclusion is responsible for the velocity enhancement. The behavior of velocity enhancement has a tendency to approach a prediction with concentration-dependent friction model, which can be found particularly within the range of $We$ less than about 2.5. Above this $We$, however, the model curve may display a somewhat sudden increase in velocity enhancement factor than the experimental observation. We recall mostly that this disagreement could be attributed to the onset of restriction on the finite chain extensibility as a flow strength increases indeed. Subsequently, it results in the limitations on the deformation of polymer chain.

In case of anionic HPAM, the relaxation time was estimated from the reported data on the intrinsic viscosity with different NaCl concentrations and the approximated chain extension relationship (Chun and Park, 1994). The $\chi$ versus $We$ plot for HPAM shown in Fig. 9 displays a considerable discrepancy between the measured velocity enhancement factor and the model prediction. This behavior is obviously different from what is observed for nonionic PAAm. Noticeable results, in which the experimental data for HPAM show an earlier increase of velocity enhancement than that of the present model, allow us to consider the contribution of colloidal force between charged wall and charged polymer.

Contrary to the Fig. 8, a single curve can not be acquired in HPAM due to the different increasing slopes of the velocity enhancement according to an eluant ionic strength shown in Fig. 9. The implication of different slopes provides an information on the behavior of chain extension with shear rates. As the polymer chain becomes more flexible, the slope increases more sharply with shear rates. It is important to emphasize that a special care needs in the fractionation of charged flexible polymer. Furthermore, it may be undesirable in the application of CHDF for size analysis since the elution time with high ionic strength strongly depends on the flow rate. Without a priori how the elution time is affected in a capillary column, the measurement of elution time would reveal little about the size. The optimum
Fig. 9 Comparison between experimental results and model prediction for the velocity enhancement factor versus $W_e$ relation in HPAM with various NaCl concentrations; relaxation times approximately estimated with the assumption are 0.38, 0.10, and 0.02 msec for NaCl concentrations of none, 0.1, and 100 mM, respectively.

Conclusion

The velocity enhancement behavior of flexible polymer has been observed by performing the CHDF flow experiments with water-soluble polymers. We considered an appropriate model for the flexible chain based on the conformation-dependent friction, and the radial profile of chemical potential could be provided. The diffusion theory was applied to predict unambiguously the flow-induced polymer migration, in which the concentration depletion was controlled by the flow strength. In case of the PAAm, it can be noted that the agreement between experiments and model prediction is found for the $W_e$ less than 2.5. However, our prediction for the higher $W_e$ show a somewhat sudden increase in velocity enhancement factor than the experimental observation. The anionic HPAM did not formulate a single curve between $\chi$ and $W_e$ because of the different increasing rates of velocity enhancement with the eluant ionic strength. It means the slope of velocity enhancement on the shear rate is fairly related to a change of chain conformation regarding the eluant ionic strength. The contribution of colloidal force has been observed from the velocity enhancement of HPAM. In this, the concentration depletion will become pronounced evidently, which results from the enhanced racial migration incorporating with electrostatic repulsion.

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Nomenclature

- $a$ = subunit length [nm]
- $C_0$ = polymer concentration [g cm$^{-3}$]
- $C_{w}$ = critical overlap concentration [g cm$^{-3}$]
- $C_{b}$ = polymer concentration at the bulk region [g cm$^{-3}$]
- $C_{e}$ = initial polymer concentration at capillary entry [g cm$^{-3}$]
- $D$ = polymer mass diffusivity [cm$^{2}$ sec$^{-1}$]
- $f$ = entropic chemical potential [erg]
- $J$ = diffusion flux [g cm$^{-1}$ sec$^{-1}$]
- $kT$ = Boltzmann temperature [erg]
- $L$ = end-to-end distance [em]
- $M_w$ = average molecular weight [g mol$^{-1}$]
- $N$ = number of subunits
- $Q$ = friction factor ratio
- $R_0$ = gas constant [em$^{2}$ sec$^{-2}$ rool$^{-1}$ X$^{-1}$]
- $r_i$ = capillary inner radius [em]
- $r$ = radial position [em]
- $t$ = time [sec]
- $V$ = polymer velocity vector [cm sec$^{-1}$]
- $<V_e>$ = average fluid velocity [cm sec$^{-1}$]
- $<V_p>$ = average polymer velocity [cm sec$^{-1}$]
- $W_e$ = dimensionless Weissenberg number (=t,$\eta$)
- $x_i$ = coordinates of polymer molecule [em]
- $x$ = dimensionless end-to-end distance
- $x_0$ = separation vector
- $z$ = axial position [em]

Subscripts

- $m$ = ratio of mean separation vector
- $p$ = position-dependent shear rate
- $w$ = wall shear rate
- $\eta$ = Kramers delta
- $e$ = friction factor
- $\eta_s$ = intrinsic viscosity [em$^{2}$ g$^{-1}$]
- $\eta_v$ = solution viscosity [g cm$^{-1}$ sec$^{-1}$]
- $\lambda$ = relaxation time [sec]
- $\chi$ = velocity enhancement factor
- $\psi$ = configurational distribution function of polymer chain
- $\Omega$ = constant associated with relaxation time

Superscripts

- $*$ = dimensionless variable

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