Simulation of enhanced power generation by reverse electrodialysis stack module in serial configuration

Kwang Seok Kim, Won Ryoob, Myung-Suk Chun, Gui-Yung Chung

Complex Fluids Laboratory, National Agenda Research Division, Korea Institute of Science and Technology (KIST), Seongbuk-gu, Seoul 136-791, Republic of Korea
Department of Chemical Engineering, Hong-Ik University, Mapo-gu, Seoul 121-791, Republic of Korea

HIGHLIGHTS

• Simulations of reverse electrodialysis (RED) stack module in serial configuration
• Orthogonal collocation on finite element solution to convection–diffusion problem
• Unique implication of Legendre polynomial and Gaussian quadrature integration
• Performance evaluations and validity of the module from power and energy densities
• Adverse ion flux can occur with condition of extremely low Peclet number.

ABSTRACT

By extending the theoretical model previously formulated for the single unit problem, we fully analyze the performance of the reverse electrodialysis stack module aiming for electric power generation. In our prototype, each single unit is assembled in serial, where the mixing is allowed in connecting channels to exclude the ionic polarization. The numerical algorithm accompanying the orthogonal collocation on finite element method is applied with Legendre polynomial and Gaussian quadrature integrations to predict the ion concentration profile in each compartment, power, energy, and corresponding current densities. As the number of units increases from 1 to 8, the maximum power and the maximum current densities decrease from 9 to 1 mW/m² and from 3.6 to 1.3 A/m², respectively, but the maximum energy density increases from 1.5 to 4 mW h/m³. Pursuing the justification of the validity of our RED stack module, we determine the unique compartment thickness that makes each density maximum for the specified number of units. Power and current densities increase with increasing characteristic fluid velocity, while the energy density maintains constant (ca. 0.36 mWh/m³) prior to monotonic decreasing at Pe > 70.

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1. Introduction

Reverse electrodialysis (RED), or sometimes called electrodialysis reversal, is one of the reverse desalination processes, and recently noticed that electrical energy can be produced through the salinity gradients [1–10]. This principle can be also applied in reactivation of filtration membranes by removing fouling deposits [11]. The initial attempt to materialize the thermodynamic concept can go back to 1950s [12], and the success has been given consistent spotlights due to its potential to be a good alternative and sustainable energy source. Nowadays, the advancement of the RED process results in the hybrid system with the seawater desalination units [13], the pressure retarded osmosis [14], and the microbial power cell [15].

There are many efforts to maximize the performance and efficiency of the energy harvesting in the RED by avoiding power loss [5] or by expanding module dimension [6,7]. It should be pointed out that power density is a key performance to evaluate the RED as an established process. Therefore, further studies are required to enhance the power density, inter alia, by designing more innovative stack system [9,10]. For enhancing the power generation, the effect of time-periodic pulsatile counter-current flow has been investigated by simulating the single RED unit [16]. Due to the polarization of the ions near the ion-exchange membrane surface, the voltage drop was observed before the flow gets into the steady state, which is consistent with the experimental results [2].

In this study, we develop the numerical framework capable of evaluating of performance for the RED stack module in serial configuration and with mixing between two consecutive compartments, which were not elaborated in previous studies. The module dimension is mainly
decided by the number of units and the compartment thickness. The ion transport within compartment is analyzed by using the convection–diffusion model with the slit laminar counter-current flow. The orthogonal collocation on finite element (OCFE) method applied here to generate the mesh grid is known to be superior to the finite difference method in the aspects of the convergence, the numerical stability, and the computational load [17]. Special benefit lies on the settlement of the stability problem that is stemmed from the discontinuity of the solution. As the process parameters, we change the values of the number of units, the compartment thicknesses, and the characteristic fluid velocities to examine the behaviors of the power density, the energy density, and the current density. Although the adoption of the serial connection and the mixing should be a challenging issue, its full practical consideration is a beyond the scope of this study.

2. Theoretical considerations

2.1. The RED stack module in serial configuration

The stack sequence of the conventional RED module can be easily found in the literatures [5–8]. The prototype of the module studied here consists of multiple sheets of cation-exchange membranes (CEMs) and anion-exchange membranes (AEMs) with the length $L$ and the width $W$, denoted in Fig. 1. They are placed in the alternating order, and both ends of the module are equipped with the anode and the cathode plates. In Fig. 2, the compartment thickness $\delta_C$ is much less than the length and the width, indicating a configuration of the slit channel.

As shown in Fig. 1, one unit is designed to comprise a pair of compartments: one is the saline compartment and the other is the fresh compartment. The color arrows denote connecting channels between the compartments, where mixing is accomplished for the reduction of the ionic polarization. The saline water is supplied to the bottom of the saline compartment in the unit 1, and leaves it through the top, losing sodium cations and chlorine anions to the adjacent fresh compartments. Fresh water can be supplied to the module in two ways: being introduced to the unit 1 (as illustrated in Fig. 1) and to the last unit (not shown here). In the former case, the inlet is placed at the top of the compartment to form a counter-current flow. In the latter case, however, the position of the fresh inlet depends on the number of the units $N$, i.e., if $N$ is even, the inlet is at the bottom; otherwise, it is at the top. This study has mainly focused on the computations for the case where the fresh water is supplied to the unit 1. The other case is addressed briefly for comparison.

2.2. Transport analysis in RED unit

The saline and fresh streams switch their directions (upward ↔ downward) in the next compartments. To avoid any confusion, we set the Cartesian coordinates such that the origin is placed at the bottom in the middle of the unit, the $x$-axis is toward the fresh compartment and the $z$-axis toward the top (see Fig. 2), regardless of the flow directions. Consequently, the $y$-axis goes down the paper. According to a reasonable assumption of neglecting membrane thickness $\delta_M$ [16], the computational domain is defined as $-\delta_C < x < \delta_C$ and $0 < z < L$.

Since the diffusion of anion (e.g., $\text{Cl}^-$) takes place around 1.5 times faster than that of cation (e.g., $\text{Na}^+$) [18], the transport of cation is the rate-controlling mechanism and can consistently represent the entire species balance. Thus, the “ion” hereafter in this paper refers to the
\[
\frac{\partial c_i}{\partial t} + \mathbf{v}_i \cdot \nabla c_i = D \nabla^2 c_i. \tag{1}
\]

Here, \(c_i\) and \(\mathbf{v}_i\) are the ion concentration and the fluid velocity vector, respectively, in the compartment \(i\) at the \(j\)-th unit (\(i = s\) for saline or \(i = f\) for fresh). The diffusivity of ion \(D (= 1.33 \times 10^{-5} \text{ cm}^2/\text{s})\) is set constant. Applying the steady-state transport and the slit geometry shown in Fig. 2 to Eq. (1), the governing equation can be obtained as

\[
D \frac{\partial^2 c_i}{\partial x^2} - \mathbf{v}_i \cdot \frac{\partial c_i}{\partial x} = 0 \tag{2}
\]

where \(\mathbf{v}_i\) is the \(z\) component of the vector \(\mathbf{v}_i\). It is hypothesized in Eq. (2) that the diffusion is dominant in the lateral (\(x\) direction) ion transport within compartment and the convection is governing the streamwise (\(z\) direction) transport.

There is no flux of ion at the surface of the AEM (i.e., \(x = -\delta_c\) or \(\delta_c\)), therefore, the Neumann condition is assigned as

\[
\left( \frac{\partial c_i}{\partial x} \right)_{x = -\delta_c} = \left( \frac{\partial c_i}{\partial x} \right)_{x = \delta_c} = 0. \tag{3}
\]

We need to impose two boundary conditions at both sides of the CEM surface (i.e., \(x = 0^-\) for left side and \(0^+\) for right side). Each condition can be derived by applying ion flux continuity and on the basis of the pseudo-steady transfer across the CEM, as follows

\[
\left( \frac{\partial c_i}{\partial x} \right)_{x = 0^-} = \left( \frac{\partial c_i}{\partial x} \right)_{x = 0^+}. \tag{4}
\]

The velocity profile in the compartment can be independently obtained by solving the Navier–Stokes momentum equation for an incompressible laminar flow in the slit channel with no-slip boundary conditions. One obtains each form of the parabolic type velocity in the \(j\)-th compartments for saline and fresh water, respectively

\[
\mathbf{v}_i|_{x = 0^-} = \mathbf{v}_i|_{x = 0^+} + \left( \frac{KD}{D_{\text{salt}}/\delta_c} \right) \left( \frac{\partial c_i}{\partial x} \right)_{x = 0^+}. \tag{5}
\]

Here, \(D_{\text{salt}}\) is the diffusivity of ion through the CEM, and \(K\) is the partition coefficient of ion between water–CEM, which is assumed 0.1 [16].

Due to the mixing allowed in the connecting channel, the ion concentration at each entrance is uniform, corresponding to the mean value at the exit of the previous compartment. With these identities, we obtain each ion concentration that is addressed in the saline compartments and in the fresh compartments, respectively

\[
\mathbf{v}_i|_{x = 0^-} = \delta_c^{-1} \int_{-\delta_c}^{0} \mathbf{v}_s \left|_{z = 0^-} \right. \, dx, \tag{6}
\]

\[
\mathbf{v}_i|_{x = 0^-} = \delta_c^{-1} \int_{0^-}^{\delta_c} \mathbf{v}_f \left|_{z = 0^-} \right. \, dx. \tag{7}
\]

Here, the characteristic fluid velocity \(v_0\) is equivalent to \((- \Delta p \delta_c^{-2}) / (2 \mu)\) with the pressure drop \(\Delta p\) between inlet and outlet of each compartment and the fluid viscosity \(\mu\).
Eqs. (2)–(9) can be rewritten in the dimensionless form with the dimensionless variables, \( X = x/\delta \), \( Z = z/L \), \( C_j = c_j/c_0 \), and \( V_j = v/v_0 \), where \( c_0 \) is the saline feed concentration. The resulting equations are:

\[
\frac{\partial^2 C_j}{\partial X^2} - \text{Pe} \frac{\partial C_j}{\partial Z} = 0.
\]

(10)

\[
\left( \frac{\partial C_j}{\partial X} \right)_{X=0} - \left( \frac{\partial C_j}{\partial X} \right)_{X=1} = 0.
\]

(11)

\[
\left( \frac{\partial C_j}{\partial X} \right)_{X=0} = \left( \frac{\partial C_j}{\partial X} \right)_{X=1},
\]

(12)

\[
C_j^{(i-1)}|_{Z=1} = \int_{-1}^{0} C_j^{(i-1)}|_{Z=0} dX.
\]

(13)

\[
V_j = (-1)^i (X^2 + X).
\]

(14)

\[
\phi(X) = \frac{RT}{F} \ln \left( \frac{c_j|_{X=1}}{c_j|_{X=0}} \right) = \frac{k_B T}{F} \ln \left( \frac{c_j|_{X=1}}{c_j|_{X=0}} \right).
\]

(18)

Here, \( F \) is the Faraday constant \((= 96,485 \text{ C/mol})\), \( R \) is the universal gas constant \((= 8.314 \text{ J/mol K})\), \( k_B \) is the Boltzmann constant \((= 1.38 \times 10^{-23} \text{ J/K})\), \( z \) is the elementary charge \((= 1.6 \times 10^{-19} \text{ C})\), \( T \) is the absolute temperature, and \( \Lambda \) is the valence of ion \((= +1 \text{ for Na}^+)\). The permselectivity \( \alpha \) is assumed 1 based on the literature \([20, 21]\), which means that the membrane perfectly rejects the penetrated anions.

The ion flux across the CEM, \( J_c(z) \), is given as

\[
J_c(z) = -\frac{D_M}{\delta M} \left( c_j|_{X=0} - c_j|_{X=1} \right).
\]

(19)

Multiplying Eq. (19) by \( F \) and the specified membrane area \((W \Delta z)\) results in the electric current

\[
I(z) = -\frac{FD_M W}{\delta M} \left( c_j|_{X=0} - c_j|_{X=1} \right) \Delta z.
\]

(20)

If Eq. (20) is multiplied by only \( F \), one can evaluate the current density \( I_0 \) \((\text{A/m}^2)\). Then, the power \( P \) is the product of \( \phi \) and \( I \):

\[
P(z) \equiv \phi I = -\frac{D_M W R T}{\delta M} \left( c_j|_{X=0} - c_j|_{X=1} \right) \ln \left( \frac{c_j|_{X=1}}{c_j|_{X=0}} \right) \Delta z.
\]

(21)

The total power generated in the \( j \)-th unit is computed by integrating Eq. (21) with respect to \( z \):

\[
P_j = \int_0^1 P(z) dz = -\frac{D_M W R T}{\delta M} \left[ \left( c_j|_{X=0} - c_j|_{X=1} \right) \ln \left( \frac{c_j|_{X=1}}{c_j|_{X=0}} \right) \right].
\]

(22)

It is worth noticing that, in view of Ohm’s law, dividing Eq. (18) by \( \phi / \text{ed membrane area} \((W)\) results \((\text{A/m}^2)\). The membrane resistance was experimentally measured as a function of NaCl concentration \([22]\), and the previous results \((\text{cf. Fig. 4 therein})\) presented a similar trend expected by Eq. (24).

2.4. Power density and energy density in stack module

The purpose of the RED stack module in serial configuration is to maximize the power generation, which comes with the economical aspects, such as the installation, operation, and maintenance costs. The present study stresses on the physical point of view. The cost involved here, thus, is related to the overall working area \((= \text{NWL})\). With the above constraint, the power density \( P_d \) is defined as the total amount of power \( \Sigma P \) divided by the overall working area

\[
P_d = -\frac{D_M W R T}{\text{NWL}} \sum_{j=1}^N \int_0^1 \left( c_j|_{X=0} - c_j|_{X=1} \right) \ln \left( \frac{c_j|_{X=1}}{c_j|_{X=0}} \right) dz.
\]

(25)

\begin{table}[h]
\centering
\caption{Elements of \( 7 \times 7 \) matrices for the first \( (A_i) \) and the second \( (B_i) \) derivatives used in the OCFE with the 5th-order Legendre polynomial.}
\begin{tabular}{|c|ccccc|}
\hline
\( i \) & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
1 & -31.0 & 34.7 & 5.03 & 2.13 & -1.50 & 1.71 & -1.0 \\
2 & -12.1 & 10.1 & 3.88 & -1.50 & 0.99 & -1.10 & 0.64 \\
3 & 3.73 & -7.63 & 1.52 & 3.41 & -1.90 & 1.94 & -1.10 \\
4 & -1.88 & 3.37 & -4.00 & 0.00 & -4.00 & -3.37 & 1.88 \\
5 & 1.10 & -1.94 & 1.90 & -3.41 & -1.42 & 1.52 & 7.63 & -3.73 \\
6 & -0.64 & 1.10 & -0.99 & 1.50 & -3.88 & -1.01 & 13.1 \\
7 & 1.00 & -1.71 & 1.50 & -2.13 & -5.03 & -3.47 & 31.0 \\
\hline
\end{tabular}
\end{table}
Another way to access the performance is to calculate the energy density $E_D$. To get this, we divide the total amount of power $\Sigma P$ by the characteristic flow rate of the fresh feed ($=\dot{V}_0 W_{V0}$), such that

$$E_D = -\frac{D_R RT}{\dot{V}_0 W_{V0}} \sum_{m=1}^{N} \Phi_m \left( \frac{c_j}{c_i} \right)_{x=0} \ln \left( \frac{c_j}{c_i} \right)_{x=0} \, dz. \tag{26}$$

It is useful to understand that the energy density is the energy generation (Wh) per unit volume of fresh water.

3. Methods and input parameters for computation

3.1. Computation scheme

Solving Eq. (10) subjected to Eqs. (11)–(17), the OCFE method is applied into the generation of the 2-dimensional mesh grids [17] and the approximation of the differential or integral terms in Eqs. (10)–(15). In the simulation, the use of 3 finite elements and 7 collocation points provides the sufficient convergence in the concentration profiles. The set of the collocation points is $\Phi_C = \{\phi_m\}$ with $\phi_1 = 10^{-20}$, $\phi_2 = 0.0469$, $\phi_3 = 0.2308$, $\phi_4 = 0.5$, $\phi_5 = 0.7692$, $\phi_6 = 0.9531$, and $\phi_7 = 1$. Note that $\phi_m$ to $\phi_6$ are the zeros of the 5th-order Legendre polynomial [23] that is compressed by a factor 2 and then shifted to the right by 1/2.

In the OCFE, the column matrix for the ion concentration $C$ is approximated by

$$C \equiv Q \cdot a \tag{27}$$

where the $7 \times 7$ square matrix $Q = \{Q_{mn}\} = \{\phi_{m-1}^{n}\}$ and $a = \{a_m\}$ is the row matrix of coefficients. Thus, the 1st and the 2nd derivatives of $C$ can be given by

$$C' \equiv \Gamma \cdot a, \tag{28}$$
$$C'' \equiv \Theta \cdot a \tag{29}$$

where $\Gamma = (\Gamma_{mn}^m) = [(m-1)\phi_{m-2}^m]$, and $\Theta = (\Theta_{mn}) = [(m-1)(m-2)\phi_{m-3}^m]$ with $7 \times 7$ square matrices. Multiplying both sides of Eq. (27) by the inverse matrix $Q^{-1}$ and substituting the resulting equation into Eqs. (28) and (29) lead to $C' = (\Gamma \cdot Q^{-1}) \cdot C = A \cdot C$ and $C'' = (\Theta \cdot Q^{-1}) \cdot C = B \cdot C$. Here, the elements of the $7 \times 7$ matrices $A$ and $B$ are given in Table 1.

The integration of the concentration in each unit is accomplished using the Gaussian quadrature [17], such that

$$\int_0^1 C \, dx = \omega \cdot C \tag{30}$$

where each element of the weighting row matrix $\omega$ ($= \omega_1 = 0$, $\omega_2 = 0.118$, $\omega_3 = 0.239$, $\omega_4 = 0.284$, $\omega_5 = 0.239$, $\omega_6 = 0.118$, and $\omega_7 = 0$) is multiplied with $C$. The algorithm for solving and evaluating the equations is programmed in Mathematica (Wolfram Research, Inc., IL), and run on the 2.8 GHz Intel Core i5 CPU.

3.2. Experimental parameters and condition

For simulations in this study, the Neosepta® CMX and AMX membranes (Tokurama Soda Inc., Japan) are considered, which are widely used in electrodialysis processes and are designed to have high mechanical strength. Table 2 summarizes their detailed characteristics.
based on the previous report [22]. The membrane diffusivity of CMX is estimated as $1.94 \times 10^{-6}$ cm$^2$/s, from Eq. (23) and the membrane resistance.

The ion transports induce the electric current via redox reactions at the electrodes. The electrodes made of titanium mesh coated with precious metal oxides are placed inside each endplate shown in Fig. 1, which are suitable as anode and cathode allowing current reversal. These electrode compartments are electrically connected to the galvanostat, and Ag/AgCl reference electrodes are used for the measurement of the stack voltage. The reference electrodes are placed within reservoirs that are connected to the electrode compartments via salt bridges. Detailed descriptions on the electrode system are available in the literature [3,8,10].

The working area of a single unit ($= WL$) in our prototype module is set 400 cm$^2$ with $L = W = 20$ cm. We test the model cases in the range of $\delta C$ from $10^{-3}$ to 0.5 cm and the condition of $N = 1, 2, 4, \text{and } 8$. The value of $v_0$ is less than 5 cm/s, indicating a fact that the Reynolds number $Re (= \rho v_0 \delta C / \mu)$ does not exceed about 250.

4. Results and discussion

For the salinity of saline feed, we apply a 35 g/L NaCl aqueous solution, corresponding $c_0 = 0.599$ mol/L when a complete dissociation is assumed. In fact, specifying $c_0$ is not necessary to solve Eq. (10) because the transport properties do not depend on the ion concentration. Hence, the term “saline” is chosen to refer to the ion-donating compartment in this paper, which does not restrict the applicability of the current model within a specific range of $c_0$ (e.g., brackish water, seawater, or brine).

4.1. Ion concentration profiles

In Figs. 3 and 4, the dimensionless steady-state ion concentration in the compartments are demonstrated for $N = 4$ and $v_0 = 1.0$ cm/s, where dark blue for 0 and red color for 1 represent the fresh feed and the saline feed, respectively. The white vertical dashed lines inserted between the adjacent compartments are either CEMs or AEMs, depending on their positions.

Fig. 4. Ion concentration profiles in the RED stack module with fresh water feeding into (a) the first unit and (b) the last unit, where $N = 4$, $v_0 = 1.0$ cm/s, and $\delta C = 0.15$ cm. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)
Fig. 3 shows the concentration profile obtained for the low Pe case ($\delta_C = 0.015 \text{ cm}, \text{Pe} = 0.85$ and Re = 1.5). The saline water is supplied to the bottom of the saline compartment in the unit 1, while the fresh water comes in at the top of the fresh compartment, underlying a counter-current flow. It should be noted that, due to the thin $\delta_C$, the saline compartment loses almost all ions at the exit of unit 1, while the fresh compartment contains the considerable amount of ions at the exit. Hence, in the unit 2, the concentration of the fresh compartment becomes higher than that of the saline one, and the ion is transferred from the fresh to the saline compartments. This reverse transfer occurs again in the unit 4, resulting in the reduction of the net power generation.

Fig. 4 shows the concentration profiles obtained for the high Pe case ($\delta_C = 0.15 \text{ cm}, \text{Pe} = 85$ and Re = 15). The RED stack module in Fig. 4a has the fresh feed at the top of the fresh compartment in the unit 1 (like Fig. 3), while the module in Fig. 4b has the feed at the bottom of the fresh compartment in the unit 4. In both cases, as the streams go through one unit and another, the concentration in the saline compartment is getting lower, but the concentration in the fresh water is getting higher. Due to the mixing, the ionic polarization is not observed at each entrance. The driving force in Fig. 4a appears high in the unit 1, and decreases as the unit number increases. On the other hand, the driving force in Fig. 4b appears moderate in all the units.

The membrane resistance $R_M$ along the streamwise $z$ direction, where $N = 4$, $v_0 = 1.0 \text{ cm/s}$, and $\delta_C = 0.15 \text{ cm}$.

Fig. 5. The membrane resistance along the streamwise $z$ direction, where $N = 4$, $v_0 = 1.0 \text{ cm/s}$, and $\delta_C = 0.15 \text{ cm}$.

4.2. Performance evaluations

4.2.1. Effect of compartment thickness

Based on the concentration profiles, the power density $P_d$ (Fig. 6), the energy density $E_d$ (Fig. 7), and the current density $I_d$ with the ion flux $J_C$ (Fig. 8) are plotted at various values of $\delta_C$ ranging from $5 \times 10^{-3}$ to 0.8 cm. For $v_0 = 1.0 \text{ cm/s}$, the Re ranges from 0.5 to 80. As the number of unit increases, both power and current densities and the ion flux decrease, but the energy density increases. These trends are related to a fact that, in the stack module of multiple units assembled with single units in serial, the ion flux in a unit gets reduced when compared to the flux in the previous one. On the other hand, since the energy density is related to the cumulative amount of the ion transfer across the membrane, an addition of units increases the energy density. According to our simulation results, the power, the energy, and the current densities show the maximum values. This feature allows us to figure out that the strong streamwise convection, i.e., high Pe, can accelerate the lateral diffusion by reducing the ionic polarization in the compartments.

Fig. 6 provides also a comparison of our results and other reported data obtained in RED stack modules of $N = 30$ with parallel configurations and $\delta_M = 0.1 \text{ cm}$. Note that $\delta_C = 0.1 \text{ cm}$ in Weinstein and Leitz [6], but $\delta_C = 0.1 \text{ cm}$ (for fresh) and 1.0 cm (for saline) in Suda et al. [2]. Although their module geometry and several factors are not exactly same as ours, the present RED module generates similar level of power density. The actual benefit from the adoption of serial configuration is expected to get the enhancement of the energy density. However, the
comparison of energy density cannot be conducted due to a lack of relevant discussion in the previous studies.

4.2.2. Effects of unit number and fresh supply position

In Figs. 6 and 7, \( \delta_C \) that makes \( PD \) maximum is larger than \( \delta_C \) that makes \( ED \) maximum. This is because the energy density is related to the amount of the fresh water consumption. \( Pe \) increases with increasing \( \delta_C \), which provides possibly the reduction of ionic polarization. However, in case of \( \delta_C > 0.04 \) cm, lots of ions will stay in the saline compartment and do not participate in the power generation until they exit the module. For the same reason, as \( N \) increases, \( \delta_C \) that makes \( PD \) maximum increases but \( \delta_C \) that makes \( ED \) maximum decreases.

As mentioned above, the RED stack module shown in Fig. 4b is more difficult to design than the module in Fig. 4a. It can be pointed out that the enhancements in \( PD \) and \( ED \) by the complicated design are not considerable. For example, the condition used in Fig. 4a results in the maximum power density for \( N = 4 \) (i.e., \( PD = 2.4 \) mW/m\(^2\), and \( ED = 0.36 \) mWh/m\(^3\)). On the other hand, \( PD \) and \( ED \) estimated with the same condition are 2.61 mW/m\(^2\) and 0.39 mWh/m\(^3\), respectively, which means only 10% enhancement.

4.2.3. Effect of flow rate

In Figs. 9 and 10, the power, the energy, and the current densities are plotted with range of \( v_0 \) from 0 to 5 cm/s. According to the literature information, \( v_0 \) values used in the experimental or pilot-scale modules are almost always less than 5 cm/s. As \( v_0 \) increases, power and current densities show the monotonic increase. The energy density shows a constant value for \( v_0 \) less than about 1 cm/s, and then monotonically decreases.

Whether to use the power density or the energy density in the device design depends on the external factors. For instance, if the membrane costs high, the module should be designed to produce the maximum power density. If the fresh water is limited, the maximum energy density becomes the key parameter. According to Fig. 6, except for the single unit, the power density displays a plateau prior to the maximum value, the width of which broadens as \( N \) increases. That means that the power density is less sensitive to the change in \( \delta_C \), comparing to the energy density. This behavior may be an advantage in terms of the convenient control of the power density.

5. Conclusions

We developed the numerical algorithm for solving the convection–diffusion model with the N–P principle to evaluate the performances of the prototype RED stack module with commercially available ion exchange membranes. Owing to the multiple units assembled by serial configuration, seawater and fresh water subsequently goes through every inherent compartment. In order to deal with the suggested multiple units, the integral of the ion concentration over the compartment thickness was provided in the discretized form.

It should be recognized that, with increasing number of units \( N \), the maximum values of the power, the current density, and the ion flux decrease, but the maximum energy density increases. While ionic polarization is reduced by the increase of compartment thickness \( \delta_C \), there exists \( \delta_C \) that makes each density maximum for various values of \( N \). With increasing characteristic fluid velocity \( v_0 \) at the given compartment dimension, power and current densities increase,
due to a reduction of ionic polarization. The optimum \( v_0 \) can be determined to 1 cm/s because a consistent decrease in the energy density begins evidently in the range \( v_0 > 1 \) cm/s. As a special feature, with a sufficiently small compartment thickness (i.e., lower \( Pe \)), the reverse transport of ion from fresh to saline compartments can occur and then reduce power and energy densities. Our numerical framework can effectively be applied to the performance evaluations and justification of the validity of the RED stack module.

**Nomenclature**

- **A**: matrix for the first derivative
- **B**: matrix for the second derivative
- **C**: dimensionless ion concentration
- **c**: ion concentration (g/L)
- **\( c_0 \)**: ion concentration in saline feed (g/L)
- **D**: diffusivity of ion (m²/s)
- **\( D_M \)**: diffusivity of ion across membrane (m²/s)
- **E_D**: energy density (mWh/m²)
- **e**: elementary charge (\( = 1.6 \times 10^{-19} \) C)
- **F**: Faraday constant (\( = 96,485 \) C/mol)
- **\( G \)**: dimensionless parameter (\( = K D_{\delta C}/D_{\delta C} \))
- **I**: electric current (A)
- **I_D**: current density (A/m²)
- **\( J_C \)**: ion flux across membrane (g/m² s)
- **K**: partition coefficient of ion between water–C EM
- **\( K_b \)**: Boltzmann constant (\( = 1.38 \times 10^{-23} \) J/K)
- **L**: length of a unit (m)
- **N**: number of units
- **\( P_D \)**: power density (mW/m²)
- **Pe**: Peclet number (\( = \nu_{DC}^2/LD \))
- **\( \rho \)**: pressure drop (Pa)
- **R**: Reynolds number (\( = \rho v_{DC} \mu/J \))
- **R_M**: membrane resistance (Ω cm²)
- **T**: absolute temperature (K)
- **V**: dimensionless fluid velocity
- **\( \nu \)**: fluid velocity (m/s)
- **\( \nu_0 \)**: characteristic fluid velocity (m/s)
- **W**: width of a unit (m)
- **X**: dimensionless lateral coordinate
- **x**: lateral coordinate (m)
- **Z**: dimensionless streamwise coordinate
- **\( z \)**: streamwise coordinate (m)

**Greek letters**

- **\( \alpha \)**: permselectivity
- **\( \delta_C \)**: compartment thickness (m)
- **\( \delta_M \)**: membrane thickness (m)
- **\( \Lambda \)**: valence of ion
- **\( \phi \)**: open circuit voltage (V)
- **\( \mu \)**: viscosity of fluid (g/cm s)

**Subscripts**

- **f**: fresh water
- **M**: membrane
- **n**: index for collocation point
- **s**: saline water

**References**